Longitudinal Data Analysis

methods@manchester summer school

Day 4 | afternoon session

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30/06—04/07

Using regression to estimate the difference-in-differences

Estimator (Regression 1)

We can obtain the difference in differences using regression techniques.

$$Y_i = \alpha + \beta_1 \cdot D_i + \beta_2 \cdot T_i + \delta \cdot (D_i \cdot T_i) + \varepsilon.$$

We can see that:

$E[Y_i \mid D_i, T_i]$	$T_i = 0$	$T_i = 1$	*Changes after - before*
$D_i = 0$	α	$\alpha + \beta_2$	β_2
$D_i = 1$	$\alpha + \beta_1$	$\alpha + \beta_1 + \beta_2 + \delta$	$\beta_2 + \delta$
Treated - control	β_1	$\beta_1 + \delta$	δ

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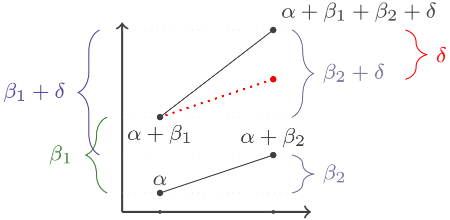

DiD: First differences estimator

Estimator (Regression 2)

With panel data we can use regression with first differences:

$$\Delta Y_i = \alpha + \delta \cdot D_i + X'\beta + u,$$

where $\Delta Y_i = Y_i(1) - Y_i(0)$.

- *With two periods this gives the same result as other regressions*

Advantages of the regression estimator

1. We can include covariates
   - Controlling for some covariates may increase precision
   - Time-varying covariates may strengthen the parallel assumptions
   - (add covariates cautiously! e.g., beware of post-treatment bias)
2. Easy to calculate standard errors
   - (though be careful about clustering)
3. Easy to extend to other types of treatment
   - (not just binary)

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Some limitations

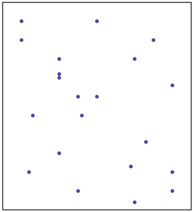
>> This setup only works for the simplest scenario with two time periods

- ⇝ It doesn't make use more periods
  - Useful to make careful assessments of time trends
- ⇝ Sometimes different units are treated at different time points

Fixed effects models

Intuition of fixed-effect regression

>> Assume a pool of structured data



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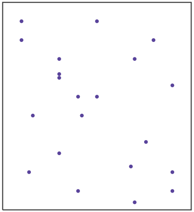

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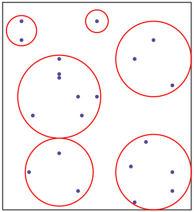

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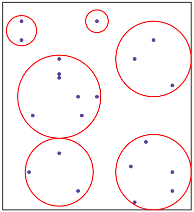

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· Pooled approach

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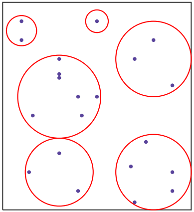

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- Pooled approach
- Between approach

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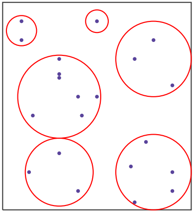

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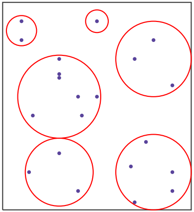

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- **Fixed Effects**

>> Focus on within-group variation

>> Implementation: dummy variables for each group j (γ_j)

$$Y_{ij} = \gamma_j + \beta \cdot X_{ij} + \varepsilon$$

>> What about panel data?

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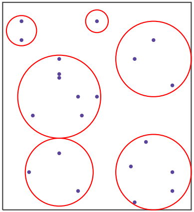

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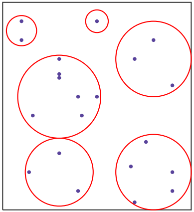

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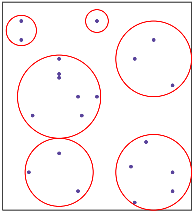

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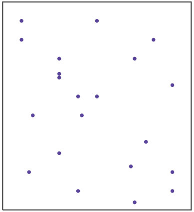

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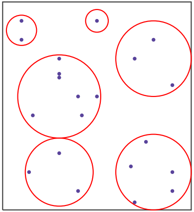

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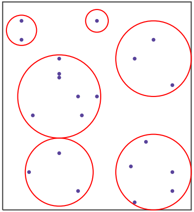

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Unit FEs and time-constant confounders

AMERICAN JOURNAL of POLITICAL SCIENCE

When Should We Use Unit Fixed Effects Regression Models for Causal Inference with Longitudinal Data?

Kosuke Imai Harvard University
In Song Kim Massachusetts Institute of Technology

Unit FEs and time-constant confounders

- Imai & Kim (2019) show that unit FEs might not be that effective in adjusting for unobserved time-constant confounders

- The issue is related to possible dynamic causal relationships

What possible dynamic relationships?

- Some dynamic causal relationships compromise unit FEs

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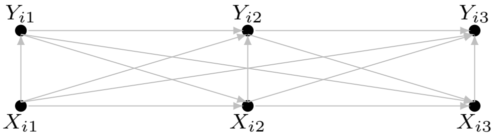

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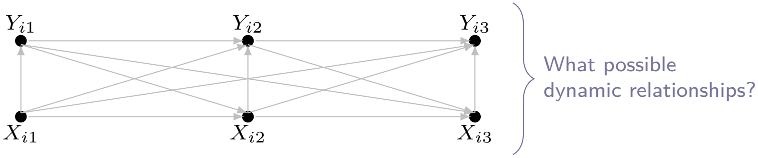

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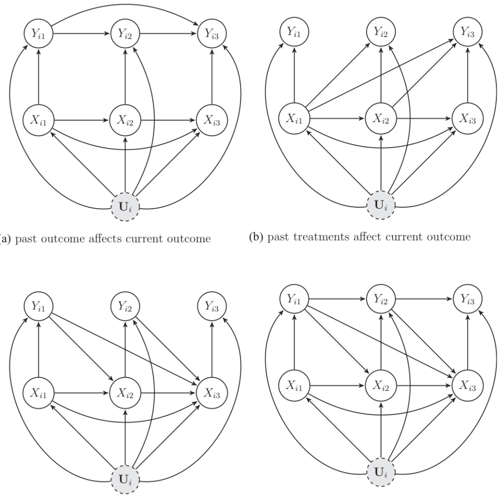
Unit FEs and time-constant confounders

FIGURE 2 Identification Assumptions of Regression Models with Unit Fixed Effects



(a) past outcome affects current outcome

(b) past treatments affect current outcome

(c) past outcomes affect current treatment

(d) past outcomes affect both current outcome and treatment

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Unit FEs and time-constant confounders

Key assumptions of unit fixed effects models

1. Past treatments do not directly influence current outcome
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>> What about time fixed-effect?

Difference-in-differences with multiple periods

DiD: Two-way fixed-effect regression

Estimator (Regression with Multiple Time Periods)

We can generalise to multiple groups/time periods using unit and period fixed-effects ('two-way' fixed-effect model):

$$Y_{it} = \gamma_i + \alpha_t + \delta \cdot D_{it} + \varepsilon$$

- γ_i *is a fixed-effect for units (dummy for each unit)*
- α_t *is a fixed-effect for time periods (dummy for each period)*
- δ *is the DiD estimate based on* D_{it}

Very flexible approach

- we can replace D_{it} with almost any type of treatment (not only binary)
- we can extend easily to multiple periods
- we can have units treated at different times
- we can estimate unit-specific time trends by including a unit-period interaction
  - $\rightsquigarrow$ useful when treatment occurs at different times for different units and there are slight deviations from parallel trends

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Why does two-way fixed-effect regression estimate the DiD?

- ⇝ Unit FEs means that we are only using *within unit* variation in Y to calculate the effect of D
  - i.e., *changes* over time!
  - This removes *all* time-constant confounders
- ⇝ Time FEs means that we remove the effect of any changes to the response variable that affect all units at the same time
- ⇝ $\hat{\delta} \rightarrow \hat{\delta}_{ATT}$ (*it might not be that simple...*)
  - It is hard to provide a visual inspection of the parallel trends assumption here as treatment may switch on at different time for different units
  - Nevertheless, we are still assuming that treated/control units would have evolved identically over time in absence of treatment

>> Why not always use unit dummies?

- Fine in panel data, as we have same units at several points in time
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Some caution with two-way fixed-effect models

Maxim Ananyev
@maximananyev

A rare photo of an applied economist keeping up with the difference-in-differences literature

Summary

Summary

- $\rightsquigarrow$ Causal inference with observational data is really hard!
- $\rightsquigarrow$ Longitudinal data can help, but it's not a silver bullet
  - Have a look at all assumptions involved
  - Parallel trends is an untestable assumption
- $\rightsquigarrow$ This is a fast-changing topic. Keep up with the literature!
  - Callaway and Sant'Anna (2020); Callaway et al. (2021); Imai et al. (2023); Goodman-Bacon (2018)
- $\rightsquigarrow$ Now let's see how to estimate those models using R!

Thank you!

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Callaway, B., A. Goodman-Bacon, and P. H. Sant'Anna (2021). Difference-in-differences with a continuous treatment. *arXiv preprint arXiv:2107.02637*.

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Imai, K., I. S. Kim, and E. H. Wang (2023). Matching methods for causal inference with time-series cross-sectional data. *American Journal of Political Science 67*(3), 587–605.